HW 1 Help

30. ORGANIZE AND PLAN We must find the average speed in m/s of a runner under three different conditions of time and distance. For each of these, we'll use the definition of average speed, $\bar{v} = \dfrac{\text{distance traveled}}{\Delta t}$. However, in each scenario, we'll have to convert time or distance, or both, to the proper units.

Known: (a) $\text{distance traveled} = 41\text{ km};\ \ \Delta t = 2\text{ h }25\text{ min};$ (b) $\text{distance traveled} = 1500\text{ m};\ \Delta t = 3\text{ min }50\text{ s};$ (c) $\text{distance traveled} = 100.\text{ m};\ \ \Delta t = 10.4\text{ s}$

SOLVE

(a) Start by converting 2 h 25 min to seconds. We think of this as $2\text{ h} + 25\text{ min}$ and convert each term separately:

$$\Delta t = 2\text{ h}\left(3600\text{ s/h}\right) + 25\text{ min}\left(60\text{ s/min}\right) = 8700\text{ s}$$

Then we convert 41 km to m,

$$\text{distance traveled} = 41\text{ km}\left(1000\text{ m/km}\right) = 41{,}000\text{ km}$$

Now we use the definition of average speed.

$$\bar{v} = \frac{\text{distance traveled}}{\Delta \text{t}} = \frac{41{,}000\text{ m}}{8700\text{ s}} = 4.7\text{ m/s}$$

(b) The distance is already given in meters. We convert the time to seconds

$$\Delta t = 3\text{ min} + 50\text{ s} = 3\text{ min}\left(60\text{ s/min}\right) + 50\text{ s} = 230\text{ s}$$

Then

$$\bar{v} = \frac{\text{distance traveled}}{\Delta \text{t}} - \frac{1500\text{ m}}{230\text{ s}} = 6.5\text{ m/s}$$

(c) Here the distance is already in meters and the time is already in seconds.

$$\bar{v} = \frac{\text{distance traveled}}{\Delta \text{t}} = \frac{100.\text{m}}{10.4\text{ s}} = 9.62\text{ m/s}$$

REFLECT The lengths of these races systematically decrease from very long (a marathon!) to moderate (just under a mile) and finally to a short dash. We can expect the average speeds to increase from a bit more than a fast walk (4.7 m/s) to a fast run (9.62 m/s).

36. ORGANIZE AND PLAN In a race, the winner is the contestant that starts at position x_0 and first reaches the finish line x. Finishing first means reaching x with a lower time Δt than the other contestant. Here we will compare the elapsed times of the two boats. The boat with the lower Δt is the winner. We will use subscripts (1) and (2) for boats 1 and 2, respectively, and a second subscript stating the first or second variable of the boat during the race.

***Known*:** distance traveled $= 2.000$ km; Boat 1: $v_{11} = 4.0$ m/s; $v_{12} = 3.1$ m/s; distance traveled$_{11} = 1500$ m; Boat 2: $v_{21} = 3.6$ m/s; $v_{22} = 3.9$ m/s; distance traveled$_{21} = 1200$ m.

SOLVE Since speed is given in m/s, we will need to convert the length of the race to meters.

$$2.000\ \text{km} \times (1000\ \text{m/km}) = 2000.\text{m}$$

Now, for boat 1, the first leg requires time

$$\Delta t_{11} = \frac{\text{distance traveled}_{11}}{v_{11}} = \frac{1500\ \text{m}}{4.0\ \text{m/s}} = 375\ \text{s}$$

The second leg of the race requires

$$\Delta t_{12} = \frac{\text{distance traveled}_{12}}{v_{12}} = \frac{2000\ \text{m} - 1500\ \text{m}}{3.1\ \text{m/s}} = 161\ \text{s}$$

The total time for boat 1 is then

$$\Delta t_1 = \Delta t_{11} + \Delta t_{12} = 375\ \text{s} + 161\ \text{s} = 536\ \text{s}$$

Doing the same calculations for boat 2,

$$\Delta t_{21} = \frac{\text{distance traveled}_{21}}{v_{21}} = \frac{1200\ \text{m}}{3.6\ \text{m/s}} = 333\ \text{s}$$

$$\Delta t_{21} = \frac{\text{distance traveled}_{21}}{v_{21}} = \frac{2000\ \text{m} - 1200\ \text{m}}{3.9\ \text{m/s}} = 205\ \text{s}$$

The total time for boat 1 is

$$\Delta t_2 = \Delta t_{21} + \Delta t_{22} = 333\ \text{s} + 205\ \text{s} = 538\ \text{s}$$

Boat 1 reaches the finish line first and wins the race.

REFLECT Another way of determining the winner is to calculate the higher average speed of each boat and to compare these values. The boat with the higher average speed wins. However, average speed is a function of total time, so this alternate method requires two additional math steps.

40. **ORGANIZE AND PLAN** This problem contrasts speed and velocity. When we calculate speed, we use the absolute distance traveled on each leg of a trip. When we want velocity, all we care about is how far we end up away from the starting point. This means we have to use the sign of each direction traveled. We'll use the subscripts (1), (2), and (3) for the three legs of the trip.

Known**:** $v_{x1} = 100.\text{ km/h}$; $\Delta t_1 = 30.0\text{ min}$; $v_{x2} = 60.0\text{ km/h}$; $\Delta t_2 = 10.0\text{ min}$; $v_{x3} = -80.0\text{ km/h}$; $\Delta t_3 = 20.0\text{ min.}$

SOLVE We must first convert the elapsed times to hours to be consistent with the velocity units.

$$\Delta t_1 = 30.0\text{ min}\left(1\text{ h}/60\text{ min}\right) = 0.500\text{ h}$$

$$\Delta t_2 = 10.\text{min}\left(1\text{ h}/60\text{ min}\right) = 0.167\text{ h}$$

$$\Delta t_3 = 20.0\text{ min}\left(1\text{ h}/60\text{ min}\right) = 0.333\text{ h}$$

$$\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3 = 0.500\text{ h} + 0.167\text{ h} + 0.333\text{ h} = 1.000\text{ h}$$

First we find the average velocity, taking into account the negative sign on v_{x3}

$$\Delta x_3 = v_{x3}\Delta t_3 = \left(-80\text{ km/h}\right)\left(0.333\text{ h}\right) = -26.6\text{ km}$$

$$\Delta x_1 = \text{distance traveled}_1 \text{ and } \Delta x_2 = \text{distance traveled}_2$$

Adding the three separate displacements,

$$\Delta x = 50.0\text{ km} + 10.0\text{ km} + \left(-26.6\text{ km}\right) = 33.4\text{ km}$$

and the average velocity is

$$\overline{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.4\text{ km}}{1.000\text{ h}} = 33\text{ km/h}$$

Then we find the total distance traveled:

$$\text{distance traveled}_1 = v_{x1}\Delta t_1 = \left(100.\text{km/h}\right)\left(0.500\text{ h}\right) = 50.0\text{ km}$$

$$\text{distance traveled}_2 = v_{x2}\Delta t_2 = \left(60.0\text{ km/h}\right)\left(0.167\text{ h}\right) = 10.0\text{ km}$$

Since we are calculating distance, not velocity, we use the absolute value of v_{x3}

$$\text{distance traveled}_3 = \left|v_{x3}\right|\left(\Delta t_3\right) = \left(\left|-80.0\text{ km/h}\right|\right)\left(0.333\text{ h}\right) = 26.6\text{ km}$$

$$\text{distance traveled} = \text{distance traveled}_1 + \text{distance traveled}_2 + \text{distance traveled}_3$$

$$\text{distance traveled} = 50.0\text{ km} + 10.0\text{ km} + 26.6\text{ km} = 86.6\text{ km}$$

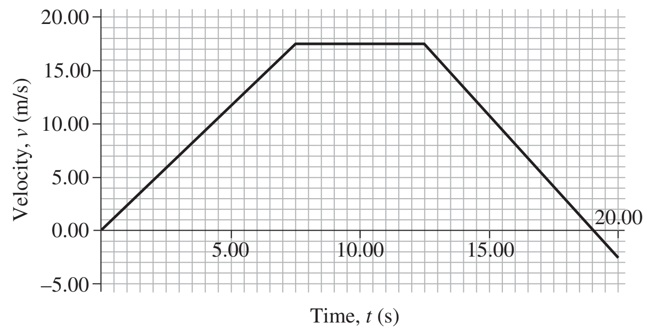
So the average speed is

$$\overline{v} = \frac{\text{distance traveled}}{\Delta t} = \frac{86.6\text{ km}}{1.000\text{ h}} = 87\text{ km/h}$$

REFLECT Displacement is never greater than distance, and is always less than distance if a change in direction takes place during the motion. Likewise, average velocity is never greater than average speed.

48. ORGANIZE AND PLAN Average acceleration is found using $\bar{a}_x = \frac{v_x - v_{x0}}{t - t_0}$. We'll read the velocity values from the vertical axis of the graph shown in Figure P2.48 in the text. Final and initial values for time come from the horizontal axis.

Known:



SOLVE A systematic way to set up this problem is to create a table. Each line of the table represents one time period. We calculate average acceleration from the values in each line. Reading time and velocity values from the graph, we obtain:

t_0, s	t, s	v_{x0}, m/s	v_x, m/s	a_x, m/s^2
0	5	0	12	2.4
5	10	12	18	1.2
10	15	18	13	–1.0
15	20	13	–3	–3.2

REFLECT This graph consists of three straight line segments connected by smooth curves. The slope of each line segment is the acceleration. However, the second and third time intervals include parts of the curves, so the average acceleration values for those intervals do not correspond to any straight line segment.

53. ORGANIZE AND PLAN The stock car's velocity is related to time by the equation $v_x = 1.4t^2 + 1.1t.$ We are asked to find v_x after $4.0\ \text{s}.$ This means we have to evaluate the given equation at $4.0\ \text{s}.$ For the units to cancel properly the value 1.4 must have units of m/s^3 and the value 1.1 must have units of m/s^2.

Known: $t_0 = 0\ \text{s};\ \ t = 4.0\ \text{s}.$

SOLVE Substituting $4.0\ \text{s}$ for $t,$ we get

$$v_x = 1.4t^2 + 1.1t = \left(1.4\ \text{m/s}^3\right)\left(4.0\ \text{s}\right)^2 + \left(1.1\ \text{m/s}^2\right)\left(4.0\ \text{s}\right) = 26.8\ \text{m/s}$$

REFLECT We know the car starts from rest at time $t = 0\ \text{s}$ since substituting this value into the given equation gives

$$v_x = 1.4t^2 + 1.1t = \left(1.4\ \text{m/s}^3\right)\left(0\ \text{s}\right)^2 + \left(1.1\ \text{m/s}^2\right)\left(0\ \text{s}\right) = 0\ \text{m/s}$$

During the time interval of this problem, velocity increases as a quadratic function of time. This situation can't last. The engine's ability to accelerate the car will decrease as engine speed increases past a certain point. Air resistance also reduces acceleration as the car moves faster.

62. ORGANIZE AND PLAN We're given acceleration and time. We have to find the sled's final speed. We can use $v_x = v_{x0} + a_x \Delta t.$ Then we can use $v_x^2 - v_{x0}^2 = 2a_x \Delta x$ to find the distance the sled has traveled.

Known: $v_{x0} = 0\ \text{m/s};\ \ a_x = 21.5\ \text{m/s}^2;\ \ \Delta t = 8.75\ \text{s}.$

SOLVE (a) First calculate the final velocity:

$$v_x = v_{x0} + a_x \Delta t = 0\ \text{m/s} + \left(21.5\ \text{m/s}^2\right)(8.75\ \text{s})$$

$$v_x = 188\ \text{m/s}$$

(b) Then we can find the distance traveled.

$$v_x^2 - v_{x0}^2 = 2a_x \Delta x$$

Rearranging,

$$\Delta x = \frac{v_x^2 - v_{x0}^2}{2a_x} = \frac{(188\ \text{m/s})^2 - (0\ \text{m/s})^2}{2\left(21.5\ \text{m/s}^2\right)} = 822\ \text{m}$$

REFLECT A velocity of $188\ \text{m/s}$ is about $420\ \text{mi/h}$. The sled travels $822\ \text{m}$ just to reach its top speed. After the end of this problem, the sled will require even more distance to slow down and stop. We would want a very long and straight path for this sled run. When we study rotational motion in Chapter 8, we'll learn more about what would happen to the sled if the path were not straight.

68. ORGANIZE AND PLAN Since we know distance and acceleration, we can use $v_x^2 - v_{x0}^2 = 2a_x \Delta x$ to find velocity. Once we have velocity, we can use the definition of acceleration to find time.

Known: $v_{x0} = 0\,\text{m/s};\quad a_x = 5.0\times10^{14}\,\text{m/s}^2;\quad \Delta x = 15\text{ cm}.$

SOLVE First, convert Δx to meters.

$$\Delta x = 15\text{ cm}\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 0.15\text{ m}$$

(a) Find the electrons' final velocity

$$v_x^2 - v_{x0}^2 = 2a_x \Delta x$$

Since $v_{x0} = 0$,

$$v_x = \sqrt{2a_x \Delta x} = \sqrt{2\left(5.0\times10^{14}\,\text{m/s}^2\right)\left(0.15\text{m}\right)} = 1.2\times10^7\,\text{m/s}$$

(b) Now find the time it takes an electron to accelerate to that velocity:

$$a_x = \frac{v_x - v_{x0}}{\Delta t}$$

$$\Delta t = \frac{v_x - v_{x0}}{a_x} = \frac{1.2\times10^7\,\text{m/s} - 0\,\text{m/s}}{5.0\times10^{14}\,\text{m/s}^2} = 2.4\times10^{-8}\text{s}$$

REFLECT Although the acceleration is very large, the electrons' speed is still only about 4% of the speed of light.

74. ORGANIZE AND PLAN Once the book leaves the student's hand, it is in free fall. In this problem we could either use the quadratic formula or find the final velocity first, followed by using the definition of acceleration. Since the final velocity is required as one of the answers, we'll choose that path.

Known: $\Delta y = -1.5\text{ m}$; $v_{y0} = 3.9\text{ m/s}$; $a_y = -g = -9.80\text{ m/s}^2$.

SOLVE (a) Finding final velocity,

$$v_y^2 = v_{y0}^2 + 2a_y\Delta y$$

$$v_y = \pm\sqrt{v_{y0}^2 + 2a_y\Delta y} = \pm\sqrt{(3.9\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-1.5\text{ m})}$$

Since the book was released above the floor and strikes the floor, we know it must be traveling in the negative direction, so we choose the negative root.

$$v_y = -6.68\text{ m/s} \cong -6.7\text{ m/s}$$

Now, knowing v_y,

$$a_y = \frac{v_y - v_{y0}}{\Delta t}$$

$$\Delta t = \frac{v_y - v_{y0}}{a_y} = \frac{-6.68\text{ m/s} - 3.9\text{ m/s}}{-9.80\text{ m/s}^2} = 1.08\text{ s} = 1.1\text{ s}$$

(b) From part (a), $v_y = -6.7\text{ m/s}$

REFLECT Solving for Δt in part (a) using the quadratic formula gives the same answer. It's just more tedious unless you have the quadratic formula programmed into your calculator! See Problem 75 for an example of using the quadratic formula in this kind of problem.

78. ORGANIZE AND PLAN In this problem, we use the acceleration due to gravity on the moon and on the Earth to compare time in free fall. Since we are given displacement, we'll use $\Delta y = v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. We don't have to solve a quadratic equation because the object starts from rest and $v_{y0} = 0\ \text{m/s}$. We'll use the subscripts "moon" and "Earth" for the values that apply only to the moon or only to the Earth.

Known: $\Delta y = -1.0\ \text{m}$; $a_{y,\text{moon}} = -1.6\ \text{m/s}^2$; $a_{y,\text{Earth}} = -9.80\ \text{m/s}^2$.

SOLVE For the moon,

$$\Delta y = v_{y0}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

Since $v_{y0} = 0$,

$$\Delta t_{\text{moon}} = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1.0\ \text{m})}{-1.6\ \text{m/s}^2}} = 1.1\ \text{s}$$

For the Earth, all that changes is the value of a_y, so

$$\Delta t_{\text{Earth}} = \sqrt{\frac{2(-1.0\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.45\ \text{s}$$

REFLECT Since acceleration on the moon is smaller than on the Earth, we expect the time in free fall to be greater on the moon. More precisely, we see that a_y is in the denominator under the radical, so we expect the two values of Δt to differ by a factor of $\sqrt{a_{y,\text{moon}}/a_{y,\text{Earth}}}$ which is what we see in the answer.

80. ORGANIZE AND PLAN The tennis-ball gun launches balls into free fall. We can use $\Delta y = v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ to solve. We'll use subscripts 1 and 2 for the first and second balls. The key is that the balls have to be in the same place at the same time at the end of the problem. For (a) we set the equations for height equal to each other and solve for Δt. For (b) we use the value of Δt from (a) and solve for Δy using $\Delta y = v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. For (c) we rearrange the definition of acceleration $a_y = \frac{v_y - v_{y0}}{\Delta t}$ to solve for the velocity of each ball.

Known: $v_{y0} = 18.5\ \text{m/s}$; $a_y = -g = -9.80\ \text{m/s}^2$; $\Delta t_1 = \Delta t_2 + 2.0\ \text{s}$.

SOLVE (a) We'll write equations of motion for each ball:

$$\Delta y_1 = v_{y0}\Delta t_1 + \tfrac{1}{2}a_y(\Delta t_1)^2$$

$$\Delta y_2 = v_{y0}\Delta t_2 + \tfrac{1}{2}a_y(\Delta t_2)^2$$

Because of the interval between balls launched by the tennis-ball gun,

$$\Delta t_1 = \Delta t_2 + 2.0\ \text{s}$$

Also, $\Delta y_1 = \Delta y_2$ at some time Δt. For ball 1, that's Δt_1; for ball 2, it's Δt_2. We set the two equations equal to each other and substitute $\Delta t_2 + 2.0\text{s}$ for Δt_1:

$$v_{y0}\Delta t_1 + \tfrac{1}{2}a_y(\Delta t_1)^2 = v_{y0}\Delta t_2 + \tfrac{1}{2}a_y(\Delta t_2)^2$$

$$v_{y0}(\Delta t_2 + 2.0\ \text{s}) - (v_{y0}\Delta t_2) = +\tfrac{1}{2}a_y(\Delta t_2)^2 - \tfrac{1}{2}a_y(\Delta t_2 + 2.0\ \text{s})^2$$

Canceling like terms,

$$(v_{y0})(2.0\ \text{s}) = -\tfrac{1}{2}a_y\left((4.0\ \text{s})\Delta \text{t}_2 + 4.0\ \text{s}^2\right)$$

Solving for Δt_2,

$$\Delta t_2 = \frac{2\left((-v_{y0})(2.0\ \text{s}) - \frac{1}{2}a_y(4.0\ \text{s}^2)\right)}{(a_y)(4.0\ \text{s})}$$

$$\Delta t_2 = \frac{2\left((-18.8\ \text{m/s})(2.0\ \text{s}) - \frac{1}{2}(-9.80\ \text{m/s}^2)(4.0\ \text{s}^2)\right)}{(-9.80\ \text{m/s}^2)(4.0\ \text{s})}$$

$$\Delta t_2 = 0.8877\text{s}$$

The time after launch of the first ball when the two balls meet is

$$\Delta t_1 = \Delta t_2 + 2.0\ \text{s} = 2.89\ \text{s} \cong 2.9\ \text{s}$$

(b) We substitute the value for Δt_1 into the equation of motion for ball 1. We use the unrounded value from (a).

$$\Delta y_1 = v_{y0}\Delta t_1 + \tfrac{1}{2}a_y(\Delta t_1)^2$$

$$\Delta y_1 = (18.8\ \text{m/s})(2.89\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.89\ s)^2 = 13\ \text{m}$$

(c) We remember that the times for the two balls are different. For ball 1,

$$v_{y1} = v_{y10} + a_y\Delta t_1 = 18.5\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.89\ \text{s}) = -9.8\ \text{s}$$

For ball 2,

$$v_{y2} = v_{y20} + a_y \Delta t_2 = 18.5\ \text{m/s} + \left(-9.80\ \text{m/s}^2\right)\left(0.888\ \text{s}\right) = +9.8\ \text{s}$$

REFLECT This is no surprise! The speed of an object in free fall is the same at a given height in its path above its launch point regardless of whether it's going up or coming down.